Multiplicity results for a class of asymptotically linear systems of second-order ordinary differential equations

Francesca Dalbono
University of Palermo, Italy

We study multiplicity of solutions to a Dirichlet problem associated with a planar system of the form

$$\begin{cases} u''(t) + A(t, u(t))u(t) = 0, & t \in [0, \pi], \\ u(0) = u(\pi) = 0, \end{cases} \tag{1}$$

where $A : [0, \pi] \times \mathbb{R}^2 \to GL_s(\mathbb{R}^2)$ is a continuous function satisfying asymptotically linear conditions.

The existence of two sign-preserving component-wise solutions is guaranteed when the Morse indexes of the linearizations at zero and at infinity do not coincide, and one of the asymptotic problems has zero-index, cf. [1].

We highlight that, under suitable extra assumptions involving the sign of the entries of the nonlinearity $A(t, x)$, the gap between the Morse indexes of the linearizations at zero and infinity provides a lower estimate on the number of solutions of problem (1), cf. [2].

Our result represents a first step in the direction of studying multiplicity of solutions to asymptotically linear Dirichlet systems in absence of a Hamiltonian structure and in absence of symmetric assumptions on the space variable. The proof is developed in the framework of topological and shooting methods. Our approach is based on conjugate points theory and on the concept of phase angles: we develop a constructive argument based on some elementary tools of phase plane analysis to determine an explicit expression of the two phase angles which allows us to visualize their reciprocal motions in the phase plane.

Acknowledgements: This work was partially supported by the PRIN project 2017JPCAPN "*Qualitative and quantitative aspects of nonlinear PDEs*", and by GNAMPA-INdAM.

[1] F. Dalbono, *Sign-preserving solutions for a class of asymptotically linear systems of second-order ordinary differential equations.* Topol. Methods Nonlinear Anal. **59** (2022), 163–191.

[2] F. Dalbono, C. Rebelo, *Multiplicity of solutions of Dirichlet problems associated with second order equations in* $\mathbb{R}^2$. Proc. Edinb. Math. Soc. (2) **52** (2009), 569–581.