$\boxed{\text{cvičení 7}}$

$\boxed{9}$ Y = počet "6" ve 100 hodech $\sim Bi\left(n = 100;\ \theta = \frac{1}{6}\right) \Rightarrow EY = 100 \cdot \frac{1}{6}$,

$DY = 100 \cdot \frac{1}{6} \cdot \frac{5}{6}$

Moivreova-Laplaceova věta s korekcí na celočíselnost:

$$\underline{P(Y \leq 20)} = P(-0{,}5 \leq Y \leq 20{,}5) = P\left(\frac{-0{,}5 - EX}{\sqrt{DY}} \leq \frac{Y - EY}{\sqrt{DY}} \leq \frac{20{,}5 - EY}{\sqrt{DY}}\right) =$$

$$= P\left(\frac{-0{,}5 - 16{,}7}{3{,}7} \leq U \leq \frac{20{,}5 - 16{,}7}{3{,}7}\right) = \Phi(1{,}02) - \Phi(-4{,}62) =$$

$$= 0{,}846 - 2.10^{-6} \doteq \underline{\underline{0{,}846}}$$

$\boxed{1}$ $X \sim Ex\left(\lambda = \frac{1}{2} \text{ min}\right) \Rightarrow f_X(x) = \begin{cases} \lambda . e^{-\lambda x} = \frac{1}{2} \cdot e^{-\frac{x}{2}} & , x \geq 0 \\ 0 & , x < 0 \end{cases}$.

$$Y = 60 + 60 . X$$

$$T(X) = 60 + 60X$$

$$T^{-1}(X) = \frac{Y - 60}{60} = \frac{Y}{60} - 1$$

$$\frac{\partial T^{-1}(y)}{\partial y} = \left(\frac{y}{60} - 1\right)' = \frac{1}{60}$$

$$\underline{\underline{f_Y(y)}} = f_X(T^{-1}(y)) . \frac{\partial T^{-1}(y)}{\partial y} =$$

$$= f_X\left(\frac{y}{60} - 1\right) . \left|\frac{1}{60}\right| =$$

$$= \frac{1}{60} \cdot \frac{1}{2} \cdot \exp\left[-\frac{1}{2}\left(\frac{y}{60} - 1\right)\right] =$$

$$= \underline{\underline{\begin{cases} \frac{1}{120} \cdot \exp\left[-\frac{y}{120} + \frac{1}{2}\right] & , y \geq 60 \\ 0 & , y < 60 \end{cases}}}$$

2. možnost – pomocí distribuční fce:

$$F_Y(y) = P(Y \le y) = P(60 + 60X \le y) = P(60X \le y - 60) =$$
$$= P\left(X \le \frac{y}{60} - 1\right) = F_X\left(\frac{y}{60} - 1\right), \quad y \ge 60 \Leftarrow x \ge 0.$$

$$X \sim Ex\left(\lambda = \frac{1}{2}\right) \Rightarrow F_X(x) = 1 - e^{-\lambda x} = 1 - e^{-\frac{x}{2}}$$

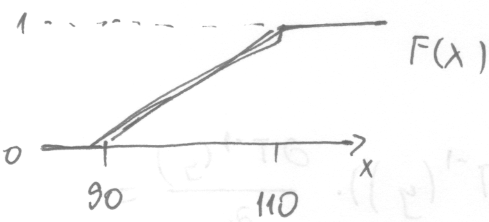
$$F_Y(y) = F_X\left(\frac{y}{60} - 1\right) = 1 - \exp\left[-\frac{1}{2}\left(\frac{y}{60} - 1\right)\right] = 1 - \exp\left[-\frac{y}{120} + \frac{1}{2}\right]$$

Hustota Y: $\underline{f_Y(y)} = F_Y(y)' = \begin{cases} \frac{1}{120} \cdot \exp\left[-\frac{y}{120} + \frac{1}{2}\right], & y \ge 60 \\ 0, & y < 60 \end{cases}$

$\boxed{2}$ $X \sim Ro(90; 110)$

$$f_X(x) = \begin{cases} \frac{1}{20}, & x \in [90; 110] \\ 0, & x < 90, \text{ nebo } x > 110 \end{cases}$$

$$F_X(x) = \begin{cases} 0, & x < 90 \\ \left(\frac{x}{20} - \frac{9}{2}\right), & 90 \le x \le 110 \\ 1, & x > 110 \end{cases}$$



$$\underline{Y = X^2 \ge 90^2} \text{ a } Y \le 110^2$$

$$\underline{F_Y(y)} = P(Y \le y) = P(X^2 \le y) = P(|X| \le \sqrt{|y|}) = P(-\sqrt{y} \le X \le \sqrt{y}) =$$
$$= F_X(\sqrt{y}) - F_X(-\sqrt{y}) = \left(\frac{\sqrt{y}}{20} - \frac{9}{2}\right) - 0 = \underline{\frac{\sqrt{y}}{20} - \frac{9}{2}}$$

$$\underline{f_Y(y)} = F_Y(y)' = \frac{1}{20} \cdot \frac{1}{2\sqrt{y}} = \begin{cases} \frac{1}{40\sqrt{y}}, & y \in [8100; 12100] \\ 0, & \text{jinak} \end{cases}$$

$$EY = \int_{8100}^{12100} y \cdot f_Y(y)\, dy = \int_{8100}^{12100} y \cdot \frac{1}{40\sqrt{y}}\, dy = \frac{1}{40} \cdot \int_{8100}^{12100} \sqrt{y}\, dy = \frac{1}{40} \cdot \left[\frac{2}{3} \cdot y^{\frac{3}{2}}\right]_{8100}^{12100} =$$

$= \frac{1}{60} \cdot \left(1331000 - 729000 \right) = \underline{\underline{10033{,}33}} \neq 100^2$!

$\overset{\shortparallel}{EX \cdot EX}$

$\boxed{3}$

x	$p_X(x)$	$y=(x-2)^2$
0	$\frac{1}{2}$	4
1	$\frac{1}{4}$	1
2	$\frac{1}{8}$	0
3	$\frac{1}{16}$	1
4	$\frac{1}{16}$	4

$$\underline{\underline{p_Y(y) = P(Y=y)}} = \begin{cases} \frac{1}{8} & , y=0 \\ \frac{5}{16} & , y=1 \\ \frac{9}{16} & , y=4 \\ 0 & , \text{jinak} \end{cases}$$

$\boxed{4}$ $X_i \sim Ro(0{,}98;\ 1{,}02)$, $i=1,2,3,4$.

(a) $P(\min(X_1, X_2, X_3, X_4) \geq 1{,}00) = P(\forall X_i \geq 1{,}00) = \left[1 - F_X(1{,}00)\right]^4 =$
$= \left(1-\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^4 = \frac{1}{16} = \underline{0{,}0625}$

(b) $P(\max(X_1, X_2, X_3, X_4) \leq 1{,}01) = P(\forall X_i \leq 1{,}01) = \left[F_X(1{,}01)\right]^4 =$
$= \underline{\underline{\left(\frac{3}{4}\right)^4 \doteq 0{,}3164}}$

$\boxed{5}$ $X \sim Ro(0;2) \Rightarrow f_X(x) = \frac{1}{2}$, $x \in [0;2]$

$Y = T(X) = \ln(1+X)$, $x \in [0;2]$

$X = T^{-1}(Y) = e^Y - 1$, $y \in [0; \ln 3]$

$\frac{\partial T^{-1}(y)}{\partial y} = e^y$

$$\underline{f_Y(y)} = f_X(e^y - 1) \cdot |e^y| = \underline{\underline{\begin{cases} \frac{1}{2} \cdot e^y & , y \in [0; \ln 3] \\ 0 & , \text{jinak} \end{cases}}}$$

6 : $X \sim N(\mu = 100,\ \sigma^2 = 15^2)$

$$\underline{P(X > 105)} = 1 - P(X \leq 105) = 1 - P\left(U \leq \frac{105-100}{15}\right) =$$

$$= 1 - \Phi\left(\tfrac{1}{3}\right) \doteq 1 - 0{,}63 = \underline{\underline{0{,}37}}$$

7 každá cesta : $\underline{EX} = \int_0^{10} x \cdot \frac{1}{10}\, dx = \frac{1}{10}\left[\frac{1}{2}x^2\right]_0^{10} = \underline{5}$

$$\underline{DX} = E(X^2) - (EX)^2 = 33{,}3 - 25 \doteq \underline{\underline{8{,}3}}$$

$$E(X^2) = \int_0^{10} x^2 \cdot \frac{1}{10} = \frac{1}{10} \cdot \left[\frac{1}{3}x^3\right]_0^{10} = \frac{100}{3} \doteq 33{,}3$$

Součet 40 cest : S_{40} $\Rightarrow$ použijeme CLV s $n = 40$

$$\underline{P(S_{40} \leq 240)} = P\left(\frac{S_{40} - 40 \cdot 5}{\sqrt{8{,}3} \cdot \sqrt{40}} \leq \frac{240 - 40 \cdot 5}{\sqrt{8{,}3} \cdot \sqrt{40}}\right) =$$

$$= \Big(\underset{\underset{N(0;1)}{\sim}}{U} \leq \frac{240-200}{\sqrt{8{,}3 \cdot 40}}\Big) = \Phi\left(\frac{40}{\sqrt{8{,}3 \cdot 40}}\right) = \Phi(2{,}195) \doteq \underline{\underline{0{,}986}}.$$

8 $Y \sim Bi(n = 2000;\ \theta = 0{,}9)$ $\quad EY = 1800$, $DY = 180$

$$\underline{\underline{P(1750 < Y < 1850)}} = P\left(\frac{1750-1800}{\sqrt{180}} < \underset{\underset{N(0;1)}{\sim}}{U} < \frac{1850-1800}{\sqrt{180}}\right) =$$

$$= P(-3{,}727 < Y < 3{,}727) = \Phi(3{,}727) - \Phi(-3{,}727) \doteq \underline{\underline{0{,}9998}}$$